

выразить всё, что ниже через $\operatorname{tg} x$

$$\sin^2 x = \operatorname{tg}^2 x / (\operatorname{tg}^2 x + 1)$$

$$\cos^2 x = 1 / (1 + \operatorname{tg}^2 x)$$

$$\sin 2x = 2 \operatorname{tg} x / (\operatorname{tg}^2 x + 1)$$

$$\cos 2x = (1 - \operatorname{tg}^2 x) / (\operatorname{tg}^2 x + 1)$$

$$\operatorname{tg} 2x = 2 \operatorname{tg} x / (1 - \operatorname{tg}^2 x)$$

$$\operatorname{tg} 3x = (\operatorname{tg} x + \operatorname{tg} 2x) / (1 - \operatorname{tg} 2x \cdot \operatorname{tg} x)$$

$$3 \operatorname{tg} 3x - \operatorname{ctg} 2x = 4 \operatorname{tg} x$$

$$3 \operatorname{tg} 3x - 1 / \operatorname{tg} 2x = 4 \operatorname{tg} x$$

$$3(\operatorname{tg} x + 2 \operatorname{tg} x / (1 - \operatorname{tg}^2 x)) / (1 - 2 \operatorname{tg} x \cdot \operatorname{tg} x / (1 - \operatorname{tg}^2 x)) - 1 / (2 \operatorname{tg} x / (1 - \operatorname{tg}^2 x)) = 4 \operatorname{tg} x$$

пусть $\operatorname{tg} x = y$

$$3(y + 2y / (1 - y^2)) / (1 - 2y^2 / (1 - y^2)) - 1 / (2y / (1 - y^2)) = 4y$$

$$3[(y - y^3 + 2y) / (1 - y^2)] / [(1 - y^2 - 2y^2) / (1 - y^2)] - [(1 - y^2) / 2y] = 4y$$

$$3[(-y^3 + 3y) / (1 - y^2)] / [(1 - 3y^2) / (1 - y^2)] - [(1 - y^2) / 2y] = 4y$$

$$3(-y^3 + 3y) / (1 - 3y^2) - (1 - y^2) / 2y = 4y \quad \checkmark$$

$$3 \cdot 2y(-y^3 + 3y) / 2y(1 - 3y^2) - (1 - y^2)(1 - 3y^2) / 2y(1 - 3y^2) = 4y$$

$$3(2y(-y^3 + 3y) - (1 - y^2)(1 - 3y^2)) / 2y(1 - 3y^2) = 4y$$

$$3(-2y^4 + 6y^2) - (1 - y^2 - 3y^2 + 3y^4) / 2y(1 - 3y^2) = 4y \cdot 2y(1 - 3y^2)$$

$$-6y^4 + 18y^2 - 1 + y^2 + 3y^2 - 3y^4 = 4y \cdot 2y(1 - 3y^2)$$

$$-9y^4 + 22y^2 - 1 = 4y \cdot 2y(1 - 3y^2)$$

$$-9y^4 + 22y^2 - 1 = 8y^2(1 - 3y^2)$$

$$-9y^4 + 22y^2 - 1 = 8y^2 - 24y^4$$

$$15y^4 + 14y^2 - 1 = 0$$

$$15y^4 + 14y^2 - 1 = 0$$

$$y^2 = t$$

$$15t^2 + 14t - 1 = 0$$

$$D = 49 + 15 = 64$$

$$t_1 = (-7 + 8) / 15 = 1 / 15$$

$$t_2 = (-7 - 8) / 15 = -1$$

$$Y^2 = 1 / 15$$

$$y = \pm 1 / \sqrt{15}$$

$$\operatorname{tg} x = 1 / \sqrt{15}$$

$$x = \pm \arctg(1 / \sqrt{15}) + \pi k$$

$$\operatorname{tg} x = 1 / \sqrt{15}$$

$$x = \pm \arctg(-1 / \sqrt{15}) + \pi k$$

$$\text{Ответ: } \pm \arctg(1 / \sqrt{15}) + \pi k; \pm \arctg(-1 / \sqrt{15}) + \pi k$$